Spectral-Element FWI: A High-Fidelity Alternative to Finite-Difference for Complex Geophysical Challenges

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Summary

The Spectral-Element Method (SEM) is a high-fidelity alternative to traditional Finite-Difference (FD) methods for seismic modelling and inversion. While FD methods are computationally efficient on regular grids, they face challenges in handling complex geological interfaces, material heterogeneities, and boundary conditions. SEM, leveraging unstructured meshes and high-order accuracy, overcomes these limitations and excels in environments with intricate topography and variable material properties. Enhanced by modern numerical techniques and GPU acceleration, SEM provides precise modelling capabilities for full-waveform modelling and inversion and advanced geophysical applications. Field data from offshore Western Australia demonstrates SEM's advantages in representing complex seabed geometries and improving waveform modelling and inversion accuracy compared to FD methods.

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Introduction

The Spectral-Element Method (SEM) is emerging as a high-fidelity alternative to Finite-Difference (FD) methods for addressing future challenges in exploration geophysics. While FD methods have long been the cornerstone of seismic waveform modelling due to their computational efficiency on regular grids, they face significant limitations in handling complex geometries, material heterogeneities, and advanced boundary conditions. In contrast, SEM combines the flexibility of unstructured meshes with high-order accuracy, making it particularly well-suited for domains with intricate topography and variable material properties. Recent advancements, such as modular implementations and optimized numerical techniques, have showcased SEM's ability to deliver exceptional precision and scalability across diverse applications, including large-scale full-waveform inversion (FWI).

By leveraging SEM, we aim to push the boundaries of seismic modelling and inversion to meet the demands of increasingly sophisticated problems in exploration geophysics, as well as emerging applications like monitoring and high-resolution near-surface characterization. In this abstract, we review the fundamental principles of SEM, highlight its advantages over FD discretization methods for solving the wave equation, and discuss our implementation strategies that make it viable for geophysical exploration applications relevant to the energy industry. We also present examples demonstrating the superior accuracy of SEM using a field dataset from Western Australia. These examples illustrate the advantages of unstructured meshes compared to rectilinear Cartesian grids in both waveform modelling and full inversion applications.

Method

The spectral element method is a high-order variant of the finite element method (FEM) that is designed to achieve greater numerical precision in solving the hyperbolic partial differential equations that govern wave propagation in complex media. Seismic waveform modelling is the backbone of computational geophysics and SEM has the potential to become a transformative tool, offering significant advantages over traditional FD methods for seismic waveform modelling. SEM leverages high-order interpolating polynomials and unstructured meshes to achieve superior accuracy and flexibility. To date SEM has been primarily employed for global and regional seismic studies (Komatitsch et al., 2002), but it recently advanced further to meet the complex demands of exploration geophysics, addressing the need for precise modelling in environments characterized by intricate topography and heterogeneous material properties (Afasaniev et al, 2019).

In our implementation, we leverage one of SEM's most defining features to utilize unstructured meshes, which provide unmatched flexibility in discretizing complex geological domains. Unlike FD methods, which are confined to structured quasi-regular Cartesian grids, or SEM on cartesian deformed meshes, that have been tested with some success in exploration scale geophysics (Trinh et al., 2017), our unstructured approach allows for seamless representation of challenging topographies such as shallow water regions, rugose offshore bathymetry and onshore terrains. These capabilities make SEM particularly effective for near-surface characterizations in areas where complex interface geometries require accurately implementing boundary conditions at acoustic, elastic or coupled media interfaces. The weak formulation inherent to SEM ensures the natural continuity of tractions and velocities across fluid-solid boundaries, enabling precise elastic wave propagation modelling in coupled media. FD methods, by contrast, require computationally expensive grid deformations to approximate these effects (Sethi et al, 2021). This advantage makes SEM an excellent choice for applications such as ocean-bottom node (OBN) acquisition, where accurate wavefield injection and propagation across interfaces are crucial.

SEM supports the direct computation of dynamic quantities such as displacement, particle velocity, strain, and stress at any point in the computational domain without interpolation. This functionality is particularly valuable for diverse geophysical applications, ranging from conventional onshore or long-offset broadband seismic exploration to emerging applications for CCS monitoring that employ fibre optic distributed acoustic sensing (DAS), ultra-high resolution (UHR) for geotechnical assessments and near-surface characterization for windfarms and geothermal energy sites. By delivering accurate surface and interface wave modelling, SEM could play a pivotal role in energy transition efforts

Practical implementation consideration

The integration of SEM using modern software-design principles (Afanasiev et al, 2019) and advanced hardware architectures have been instrumental in making it computationally viable for large scale applications. GPU acceleration has enabled significant performance gains. SEM's diagonal mass matrix characteristic facilitates efficient computations on modern highly parallel CPU and GPU computing platforms, reducing runtime and enhancing scalability.

Meshing for SEM involves discretizing the computational domain into elements that accommodate high-order polynomial basis functions, with the flexibility to use unstructured meshes that conform to complex geometries and material interfaces. The computational cost of spectral-element simulations scales directly with the number of elements in the discrete mesh, making it crucial to minimize the number of elements while maintaining accuracy. The ability of our SEM implementation to perform adaptive mesh refinement based on local wavelength plays a vital role in ensuring the practical viability of wavefield simulations. Furthermore, implementing aperture mesh cut-outs, which emulate the finite-difference method's focus on localized shot-gather domains, enables the mapping of the global meshed domain to localized meshes sufficient for computing forward and adjoint modelling for each shot-gather experiment. The inverse mapping of the adjoint simulation back to the global mesh aggregates the full gradient quantities for the given subsurface property, significantly reducing computational overhead while preserving the accuracy of the chosen model parameter updates.

In regions with confined slow velocity layers, such as soft sediments at the water bottom, SEM allows for precise matching of wavelengths. This level of refinement makes high-frequency elastic simulations feasible, which would otherwise be impossible for finite difference methods on quasi-regular FD grids, where slower velocities dictate grid sampling and time-stepping constraints. Furthermore, local time-stepping (Rietmann et al., 2015) can deliver significant speed-ups for localized low-velocity zones where a relatively small number of fine-size elements would otherwise limit the global time step of the simulation. In the most optimistic scenario, these approaches combined can result in up to an order of magnitude speed-up in both waveform and adjoint modelling while effectively constraining the frequency (f) scaling to the range f^{N+1} and f^{N}, where N is the number of spatial dimensions of the simulation domain.

Advanced non-linear optimization strategies have been seamlessly integrated into our SEM framework, including the Trust-Region Quasi-Newton L-BFGS optimizer and a novel inversion preconditioning technique based on anisotropic diffusion equations. These enhancements significantly improve convergence rates for FWI and enable semi-automated quality control of the inverted subsurface properties. The anisotropic diffusion preconditioner not only projects the model updates onto the space of geologically plausible models but also aligns naturally with SEM's mathematical formulation, and can directly operate on the unstructured meshes. Furthermore, the adoption of a novel numerical formulation for the acoustic wave equation in vertically transverse isotropic (VTI) and tilted transverse isotropic (TTI) media ensures that our SEM implementation remains versatile and effective across a broad range of computational geophysics applications from acoustic to elastic simulation and inversion. Together, these innovations establish our SEM implementation as an efficient and scalable tool for geophysical exploration at industry-relevant scales.

Field data example

We demonstrate the effectiveness of our SEM on unstructured meshes over conventional FD on rectilinear grid modelling and full-waveform inversion examples using a marine streamer field dataset from offshore Western Australia. The seismic experiment used a dual-source acquisition configuration with 25m flip-flop shooting and 12.5 m channel spacing. The survey includes 8 cables with 100 m spacing, a maximum offset of 4.6 km, and source/receiver depths of approximately 6–7 m. The survey area covers approximately 450 km^2. The seabed topography is highly complex, ranging from shallow

continental shelf areas (~200 m) to depths exceeding 1200 m, with a network of canyons and submarine mountains. This geological setting provides a benchmark for evaluating the capability of our SEM unstructured meshing to adapt to complex interfaces. To achieve this, we perform anisotropic acoustic forward modelling at 6 Hz using both FD and SEM methods, and we assess the impact of different model discretization schemes. The FD model employs a rectilinear grid with a spacing of 60 m, corresponding to 4 points-per-wavelength (ppw), which samples the wavefield and model properties. The FD discretization, depicted in Figure 1a, exhibits pronounced stair-casing artifacts (Figure 1c) at complex interfaces, potentially introducing kinematic or timing errors up to half a grid cell. In contrast, the SEM approach utilizes our unstructured meshing algorithms, which employ quadrilateral elements of polynomial order 4. These elements are sampled at 1.25 elements-per-wavelength, equivalent to approximately 5 ppw (Figure 1d, blue quadrilaterals). The meshing algorithm not only dynamically adjusts element sizes to match the local wavelength but also deforms and refines the elements to accurately sample the complex seabed topography.
The impact of these discretizations on waveform modelling is evident in the common near-channel simulation QC plots (Figures 1e and 1f). The SEM-simulated waveforms demonstrate accurate phase alignment, as indicated by the predominantly blue pattern between observed and modelled waveforms. In contrast, the FD-simulated waveforms appear less continuous and energetic, with areas of red patterns caused by phase misalignment due to poor sampling of the complex water-bottom topography.

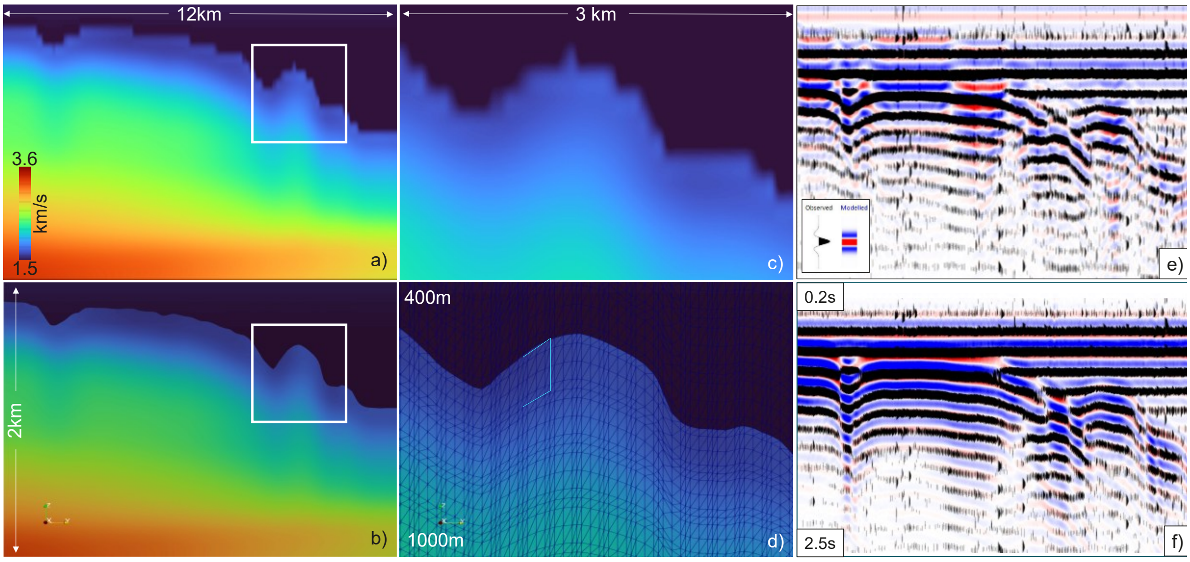

Figure 1: *Subsurface velocity models used for FD (e) and SEM (f) forward modelling calculations are shown. Model (a) is discretized using a conventional rectilinear grid with 60 m point spacing, while Model (b) is discretized using an unstructured mesh with quadrilateral elements of polynomial order 4. A zoomed-in view (white square) of the seabed topography highlights the FD (c) and SEM (d) models, with the SEM model showing the deformed quadrilateral elements. Near-channel waveforms for FD (e) and SEM (f) are computed at 550 shot locations.*

We conducted an anisotropic acoustic FWI experiment to update the initial velocity model shown in Figure 2a, using a hierarchical scheme that sweeps through five frequency bandwidth blocks from 5 to 12 Hz. Figures 2b and 2c illustrate the cumulative 12 Hz velocity updates for the FD and SEM methods, which appear broadly similar qualitatively. However, the SEM update demonstrates greater lateral continuity attributed to the anisotropic diffusion preconditioner described earlier.
To evaluate these velocity updates, we performed Kirchhoff depth migration of the recorded seismic data using both the initial model (Figure 2d) and the final FD and SEM models (Figures 2e and 2f). For comparison, the migrated seismic sections are overlaid with the respective velocity models. The red-white circle highlights a region located beneath a canyon in the seabed topography, where SEM imaging is deemed superior to FD imaging.
We hypothesize that this improvement is due to the enhanced bathymetry discretization provided by the SEM method, which reduces modelling errors, improves data misfit and, consequently, betters the

quality of the FWI velocity updates. This represents one of the first successful benchmarks of this kind in the industry.

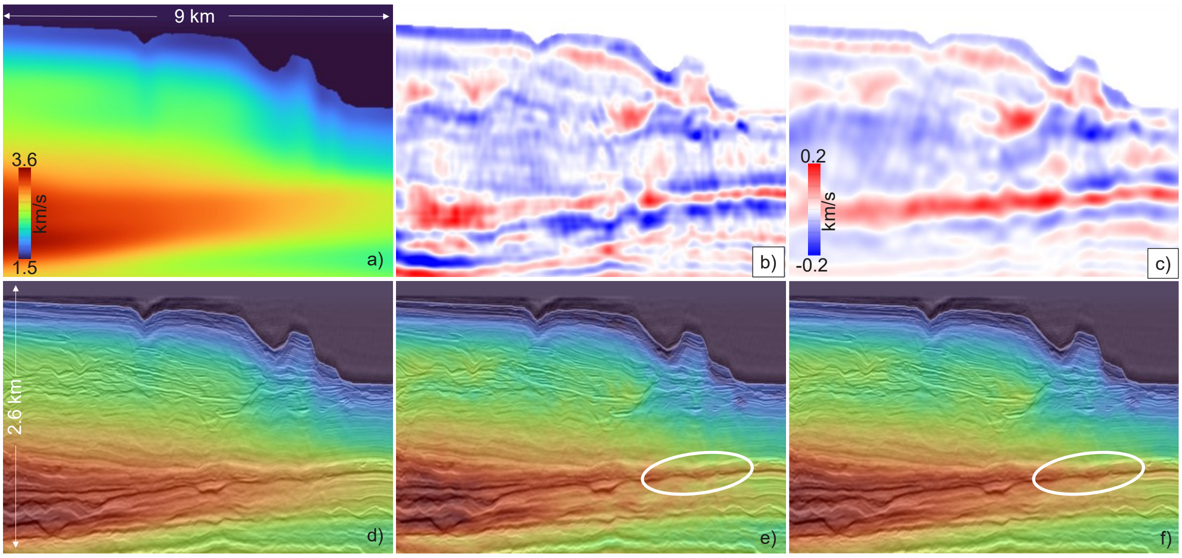

Figure 2: *Initial velocity model for the 5–12 Hz full-waveform inversion. Panels (b) and (c) show the cumulative velocity updates for FD and SEM, respectively. Panel (d) displays the initial velocity model overlaid with the Kirchhoff depth-migrated seismic data, while panels (e) and (f) present the final FD and SEM velocity models, respectively, with the migrated seismic as the background. The white circle highlights a region where SEM imaging is considered superior to FD imaging.*

Conclusion

We have demonstrated the improved accuracy of our SEM framework using a field dataset from offshore Western Australia. Our examples illustrate the advantages of SEM on unstructured meshes compared to FD on rectilinear Cartesian grids in both full waveform modelling and inversion applications. SEM represents a significant advancement in computational geophysics, offering geometrical flexibility, precision, and efficiency. By addressing the limitations of FD methods and leveraging modern computational resources, our SEM implementation will be the foundation technology for tackling the increasingly complex demands of exploration geophysics and also for applications beyond geophysics. As innovations in hardware and numerical techniques continue to evolve, SEM's role in advancing seismic modelling and inversion will only grow stronger, opening new possibilities in geophysical research and industrial applications.

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