Regularising time-domain multi-dimensional deconvolution with offset-directional derivatives

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Multi-dimensional deconvolution (MDD) is a technique used at different stages of the seismic processing and imaging value chain to suppress overburden effects by deconvolving the up- and down-going components of a given wavefield at a target of interest. Whilst the time-domain implementation has recently been identified as the de-facto solution for 2D applications, owing to its stability and ability to include physics-based preconditioners, the extension to large-scale 3D datasets is still in its infancy and may require some compromise. For example, to use a reciprocity preconditioner, one is required to solve the MDD problem for all virtual sources at once, a prohibitive scenario for 3D applications. In this work, we present a simple strategy to regularise the solution of time-domain MDD that leverages the similarity between wavefields from adjacent virtual sources. The proposed approach requires one to solve the MDD problem only for a group of virtual sources simultaneously, and therefore is amenable to 3D applications.

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Introduction

Multi-dimensional deconvolution (MDD) is a technique used in seismic processing and imaging to suppress overburden effects by deconvolving the up- and down-going components of the recorded data at a datum of interest. Although Amundsen (2001) laid its theoretical foundations more than two decades ago, it is only in the last couple of years that industrial-scale applications of MDD have emerged in the context of de-multiple of seabed data (e.g., Boiero et al. (2023), Poole et al. (2024)).

MDD can be formulated in the frequency or time domains: whilst the former allows one to solve a series of linear inverse problems (one per frequency) in an embarrassingly parallel manner, the latter has been shown to lead to a more stable deconvolution process, especially when coupled with appropriate physics-based preconditioners (e.g., causality, reciprocity, sparsity, etc.). However, applying time-domain MDD to large-scale 3D datasets comes with several practical challenges due to the extremely high computational cost of the associated modelling operator. Hong et al. (2023) have shown that one can partially alleviate these challenges by reducing the size of the kernel of the modelling operator by utilising an SVD-like compression algorithm. Nevertheless, to limit the size of the unknown wavefield and up-going data used in the inversion, MDD must be applied to a single virtual source (or a small group of virtual sources) at the time. This in turn prevents one from using robust preconditioners such as the reciprocity preconditioner, which requires inverting for all virtual sources at once.

In this abstract, we present a simple and effective approach to regularise the solution of time-domain MDD that requires to simultaneously invert for only a small group (>=3) of virtual sources. More specifically, as each virtual source gather is likely to be similar to those from nearby virtual sources, the proposed regularisation method simply minimises the difference between nearby gathers, significantly reducing incoherent artefacts that arise in the redatumed wavefield due to noise in the data and acquisition footprints. Examples on 2D and 3D datasets are presented to validate the proposed method.

Theory

Multi-dimensional deconvolution entails solving the following integral equation (Amundsen, 2001):

$$p^-(\mathbf{x}_{VS}, \mathbf{x}_S, t) = \int_{\partial D} p^+(\mathbf{x}_R, \mathbf{x}_S, t) * R(\mathbf{x}_R, \mathbf{x}_{VS}, t) d\mathbf{x}_R \quad (1)$$

where $p^-(\mathbf{x}_{VS}, \mathbf{x}_S, t)$ and $p^+(\mathbf{x}_R, \mathbf{x}_S, t)$ are the up- and down-going components of the seismic wavefield from a source $\mathbf{x}_S$ to a line of co-located receivers $\mathbf{x}_R$ and virtual sources $\mathbf{x}_{VS}$ at a given datum ∂D, and $R(\mathbf{x}_R, \mathbf{x}_{VS}, t)$ is the local reflection response deprived of overburden effects. Considering the discretised version of equation 1, a generic formulation for time-domain MDD reads as follows:

$$argmin_{\mathbf{r}} \ \|\mathbf{p}^- - \mathbf{P}^+ \mathcal{P}(\mathbf{r})\|_2^2 + \mathcal{R}(\mathbf{r}) \quad (2)$$

where $\mathcal{R}$ and $\mathcal{P}$ represent a regulariser and a preconditioner (or a chain of them), respectively, used to impose any given prior knowledge on the sought-after solution.

Let us assume that the solution $\mathbf{r}$ is composed of N_{vs} common virtual source gathers, where the N_{vs} selected virtual sources form a subset of the N_r receivers along the datum ∂D. Since one has the freedom to choose any subset of virtual sources, we will consider virtual sources that are geographically close to each other, such that their common virtual source gathers will present a high degree of similarity. In the following, we aim to devise a simple regulariser, called from here onwards the offset directional derivative (OD) regularises, that extracts traces at common offset from two nearby common virtual source gathers and minimises their squared difference (i.e., $\mathcal{R}(\mathbf{r}) = \lambda \|\mathbf{D}_{OD}\mathbf{r}\|_2^2$). Starting from the 2D scenario (Figure 1a), we consider a line of regularly sampled receivers such that the procedure required

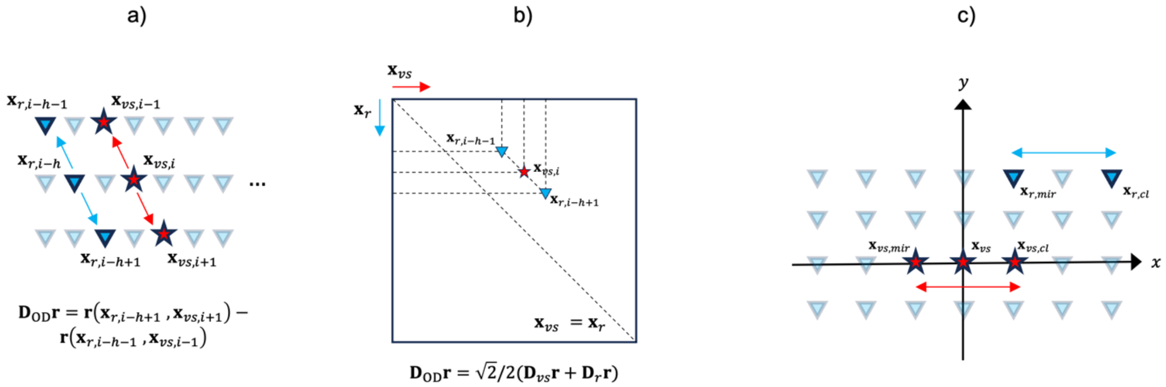

***Figure 1** Schematic representation of the process involved in the creation of the OD regulariser. a) pairing of virtual sources and receivers in a 2D geometry, b) an equivalent representation in the matrix domain with $\mathbf{D}_{vs}$ and $\mathbf{D}_r$ representing the first-order derivatives over rows and columns of the matrix, and c) pairing of virtual sources and receivers in a 3d geometry.*

to identify pairs of traces at common offset for a given virtual source $\mathbf{x}_{VS,i}$ can be described as follows:

- Identify the closest virtual source on the left ($\mathbf{x}_{vs,i-1}$) and on the right ($\mathbf{x}_{vs,i+1}$) of the chosen virtual source (note that this is not possible for the two virtual sources at the edges of the chosen subset);
- For each possible offset (i.e., $h = x_{vs} - x_r$) identify pairs of receivers with the same offset with respect to the corresponding virtual sources and define the two traces to subtract in the OD regulariser, namely $\mathbf{r}(\mathbf{x}_{r+h+1}, \mathbf{x}_{vs+1}, t)$ and $\mathbf{r}(\mathbf{x}_{r+h-1}, \mathbf{x}_{vs-1}, t)$.

Note that in the 2D scenario, the proposed regulariser is equivalent to applying a 45° directional derivative to every time slice of the reflection response (Figure 1b).

Moving on to the 3D case, let us now consider the most generic case where the receiver carpet may be irregularly sampled (note, however, that a certain degree of regularity is still needed to ensure that the spatial integral in the MDD equation is correctly evaluated). The procedure to identify pairs of traces at a common offset for a given master virtual source $\mathbf{x}_{vs}$ is composed of the following steps:

- Identify the closest virtual source ($\mathbf{x}_{vs,cl}$);
- Compute the position of the mirrored location to the closest virtual source with respect to the master virtual source: $\mathbf{x}_{mir} = 2\mathbf{x}_{vs} - \mathbf{x}_{vs,cl}$;
- Identify the virtual source closest to the mirrored location, $\mathbf{x}_{vs,mir}$ and stop if the distance between the mirrored location and the identified virtual source is beyond a given threshold;
- Re-center the receiver grids of the three of the selected virtual sources (bringing the corresponding virtual source on the origin);
- For each receiver in the master geometry, identify a receiver in the geometry of the closest virtual source and one in the geometry of the mirrored virtual source with the same source-receiver offset (or the one whose offset is the closest). If the difference of the distance between the closest and mirrored virtual sources and that of the identified receivers is below a given threshold, define the two traces to subtract in the OD regulariser: $\mathbf{r}(\mathbf{x}_{r,cl}, \mathbf{x}_{vs,cl}, t)$ and $\mathbf{r}(\mathbf{x}_{r,mir}, \mathbf{x}_{vs,mir}, t)$.

In both cases, the same procedure is repeated for all virtual sources.

Results

To begin with, the proposed regulariser is applied to a 2D line of the Volve OBC dataset. We refer the reader to Ravasi et al. (2022) for details on the pre-processing steps. Two benchmark solutions are also produced by either directly solving the MDD equation 2 with a time-window preconditioner or by

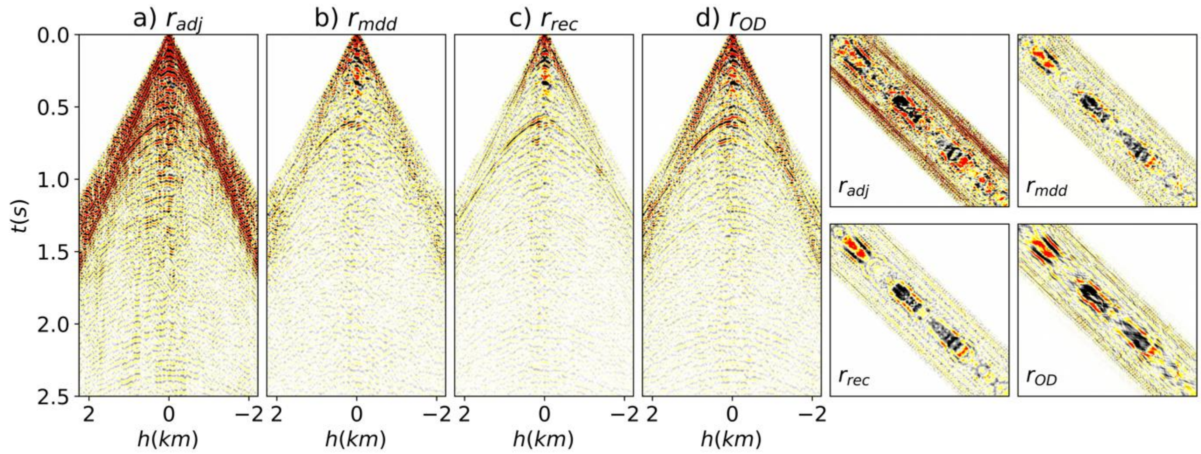

Figure 2 *Volve field data example. Virtual source gather for source in the middle of the receiver array obtained by means of a) cross-correlation ($\mathbf{r}_{adj}$), b) MDD ($\mathbf{r}_{mdd}$) without regularisation, c) MDD ($\mathbf{r}_{rec}$) with reciprocity preconditioner, and d) MDD with directional derivative ($\mathbf{r}_{OD}$). Corresponding time slices at 0.6sec are shown on the right (note that, to be able to show the entire time slice of $\mathbf{r}_{OD}$, an additional inversion is run with all virtual sources).*

chaining the time-window preconditioner to a reciprocity preconditioner (i.e., enforcing symmetry in each time slice of the local reflection response). Figures 2a and b present the redatumed wavefields obtained by cross-correlation (i.e., adjoint solution) and the first benchmark MDD approach, respectively. Since the input data presents a certain noise level, the inversion is unstable, producing a noisy estimate of the local reflection response. This result can be significantly improved by employing a reciprocity preconditioner, as shown in Figure 2c; however, as mentioned earlier, this requires solving the MDD problem for all virtual sources simultaneously, a prohibitive scenario for large-scale 3D applications. Figure 2d, on the other hand, presents the results obtained by solving MDD with the OD regularisation for a handful of virtual sources (i.e., $N_{vs} = 6$). The redatumed wavefield is comparable to that obtained using the reciprocity preconditioner. Similar conclusions can be drawn from the time slices for the four solutions.

We now consider the 3D dataset of Hong et al. (2023), which has been created from a modified version of the EAGE/SEG Overthrust model with a 300m water column added to mimic an ocean-bottom acquisition scenario. Moreover, a small amount of noise is added to the modelled data (SNR = 18 dB), and only 50% of randomly selected sources are considered in the MDD process (for a total of ~13k sources). In this case, we produce only one benchmark solution by solving the MDD equation 2 with a time-window preconditioner because applying the reciprocity preconditioner would require solving the problem for $N_{vs} = N_r$ =16k virtual sources at once. This result is compared to the one produced using the proposed OD regularisation alongside a time-window preconditioner; in this case, we solve the MDD problem for only $N_{vs} = 6$ virtual sources; this leads to the identification of ~14.5k pairs of traces whose difference is minimised as part of the regulariser (amounting to ~14% of the number of equations required to solve MDD for all virtual sources at once). The redatumed wavefields, alongside with the true reflection response, are displayed in Figure 3. Similar to the 2D example, the value of adding the proposed OD regularisation is evident.

Conclusions

We have presented a simple and effective strategy to regularise the solution of time-domain MDD, which leverages the similarity between adjacent wavefields (i.e., common virtual shot gathers from nearby virtual sources). The proposed approach, which reduces to a directional derivative in the 2D case, is amenable to 3D applications and simply requires solving the MDD problem for a group of virtual sources at once; examples on 2D and 3D datasets reveal its effectiveness, which in the 2D case is comparable to that of the reciprocity preconditioner, whilst still being easy to apply in 3D scenarios.

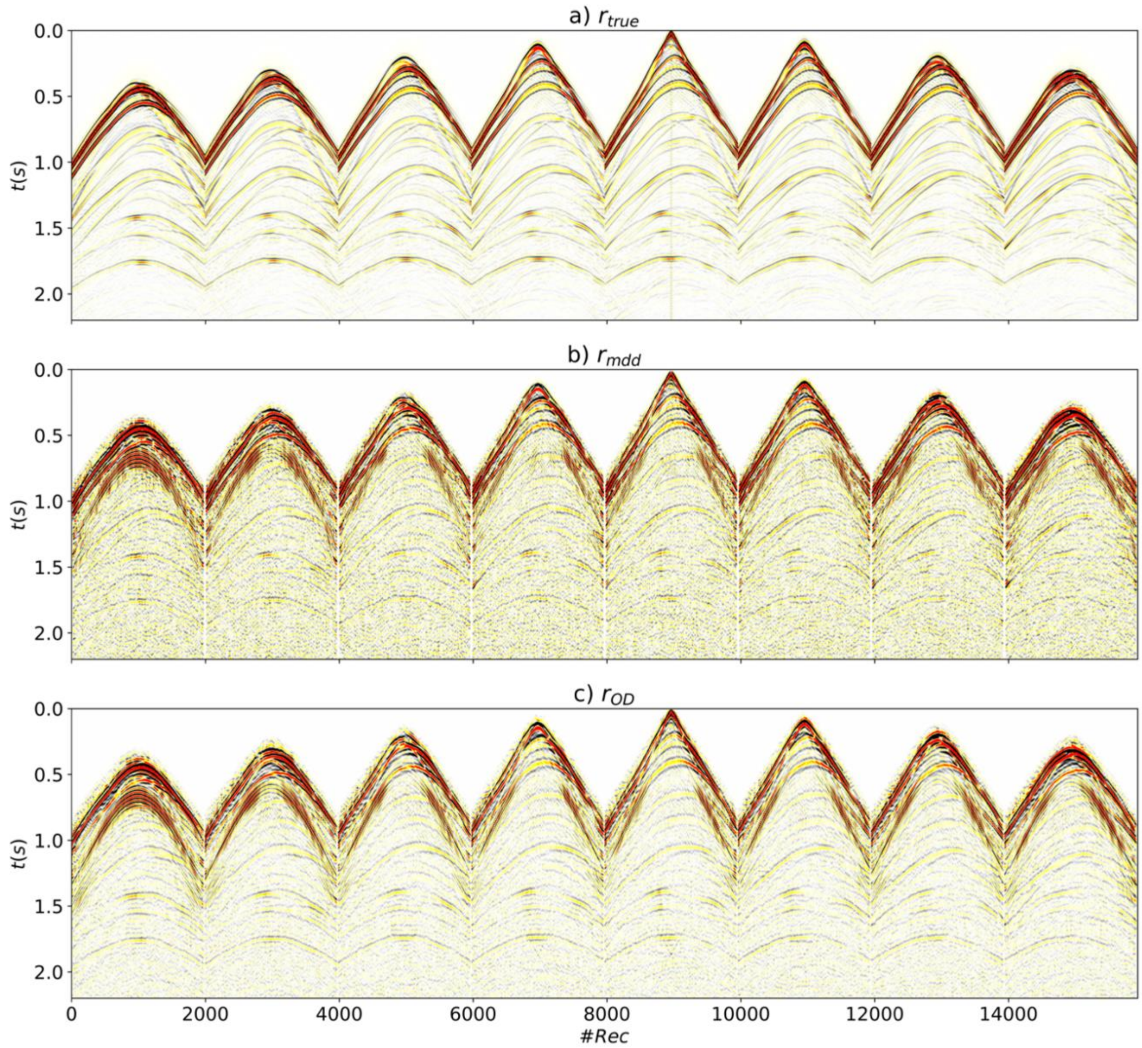

Figure 3 *EAGE/SEG Overthrust data example. a) True local reflection response and redatumed wavefields by means of a) MDD and b) MDD with directional derivative, respectively. Each panel shows the data from 8 equally spaced receiver lines for a virtual source in the middle of the receiver grid. Each receiver line is 3.5km long.*

Acknowledgements

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